Indian Statistical Institute, Bangalore
B. Math (II)
First semester 2008-2009
Semester Examination : Statistics (I)

Date: 10-12-2008 **Maximum Score 60** **Duration: 3 Hours.**

1. In a bicycle tour of 100 miles, randomly selected 15 participants travelled at the following average speeds (miles/hour).

20.2	12.0	12.4	17.6	16.5	12.6	11.1	12.3
21.1	12.5	11.4	08.1	10.8	14.7	11.0	

   (a) Draw stem and leaf diagram and box plot for the above data.

   (b) Between the box plot and the stem and leaf plot what do they tell us about the data set?

$[(3 + 5) + 3 = 11]$

2. Let $X_1,\ X_2, ..., X_n$ be a random sample from $f(x) = \theta x^{-2} I_{(\theta,\infty)}(x)$, where $\theta \in (0, \infty)$.

   (a) Check that f is indeed a *probability density function* (*pdf*).

   (b) Obtain $E(\frac{1}{X_1})$. Hence or otherwise obtain $E\left(\frac{2}{n}\sum_{i=1}^{n}\frac{1}{X_i}\right)$.

   (c) Using the results in (b) suggest an estimator for θ.

   (d) Obtain *maximum likelihood estimator* (*mle*) for θ using an appropriate version of the *pdf*.

$[2 + 3 + 2 + 4 = 11]$

3. Let T, the time required in seconds to complete 100 meter dash, for a trainee sprinter, be uniformly distributed on (θ_1, θ_2), where $0 < \theta_1 < \theta_2 < \infty$, are both unknown. Let $T_1,\ T_2, ..., T_n$ be n timings to complete 100 meter dash for the trainee.

   (a) Find the expected time required for the sprinter to complete 100 meter dash.

   (b) Hence or otherwise obtain a *method of moments estimator* for $\psi(\theta_1, \theta_2) = \frac{\theta_1+\theta_2}{2}$.

   (c) State two reasonable properties of your estimator in (b).

   (d) Obtain *maximum likelihood estimators* for θ_1 and θ_2.

$[2 + 2 + 2 + 4 = 10]$

4. The manufacturer of a certain type of automobile claims that under typical urban driving conditions the automobile will travel greater than 20 *km* per liter of petrol. The owner of an automobile of this type notes the mileages that she obtained in her own urban driving conditions when she fills the tank with petrol on 9 different occasions. She finds that the results *km per liter*, on different occasions were as follows :

$15.6, 18.6, 18.3, 20.1, 21.5, 18.4, 19.1, 20.4, 19.0.$

(a) List carefully the assumptions you must make and formulate the problem of testing of hypotheses to ascertain the manufacturer's claim. Ensure that the manufaturer's claim is part of the alternative hypothesis.

(b) Carry out a test at 5% level of significance.

(c) Find *p-value* of the test.

(d) Find *two-sided* 90% *confidence interval* for the expected distance travelled per liter of petrol.

$[2 + 6 + 1 + 3 = 12]$

5. In a large city, 200 persons were selected at random and each person was asked how many tickets she/he purchased that week in the state lottery. Suppose that the results are given in the following table. Suppose also that among the seven persons who had purchased 5 or more tickets, three persons had purchased exactly 5 tickets, two persons had purchased 6 tickets, one person had purchased 7 tickets, and one person had purchased 10 tickets.

No. of tickets purchased	0	1	2	3	4	5 or more
No. of persons	52	60	5	18	8	7

Test the hypothesis that these 200 observations form a random sample from a Poisson distribution. Also report the *p-value.*

[10]

6. Describe the *acceptance-rejection method* to draw observations from *target* density $f_Y(y)$. Establish that the method indeed results in drawing observations from *the target* density $f_Y(y)$.

$[2 + 4 = 6]$

7. Suppose we want to generate observations using *acceptance-rejection method* on random variable Y that has $Gamma(\alpha, \lambda)$ distribution with *pdf* given by $f_Y(y) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda y} y^{\alpha-1} I_{(0,\infty)}(y)$ such that $\alpha > 0$ but α is not an integer.

Let $[\alpha]$ denote the integer part of α.

(a) Can the *pdf* of $Gamma([\alpha], \lambda)$ be used as *candidate density?* Substantiate.

(b) Can the *pdf* of $Gamma([\alpha] + 1, \lambda)$ be used as *candidate density?* Substantiate.

(c) Among all *candidate densities* of $Gamma(a, \lambda)$ type, a being positive integer, find optimal value of a that minimizes the probability of rejection.

$[2 + 2 + 4 = 8]$